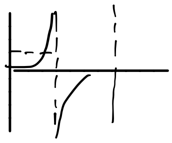
2021 ASSESSMENT MATERIALS

AS
MATHS
Trigonometry (Topic E)

Total number of marks: 39

1 State the number of solutions to the equation $\tan 4\theta = 1$ for $0° < \theta < 180°$

$0 < 4\theta < 720$



Circle your answer.

$4\theta = 45$

$\theta = 11.25$

[1 mark]

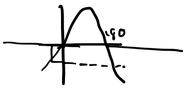
1

$\theta = 56.25$ $\quad 4\theta = 45 + 180$ $\quad 4\theta = 45 + 540$

$\theta = 101.25$ $\quad 4\theta = 45 + 360$ $\quad \theta = 146.25$

2 $\quad$ 4 $\quad$ 8

3 It is given that $\sin\theta = -0.1$ and $180° < \theta < 270°$



Find the exact value of $\cos\theta$

$\sin\theta = -0.1$, $\theta = 185.7391705$

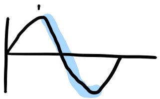
$\therefore \cos\theta = \cos(185.7\ldots)$

$= -0.999\ldots = -0.10$ (2sf)

[2 marks]

3 State the interval for which $\sin x$ is a decreasing function for $0° \le x \le 360°$

[2 marks]



from $90°$ to $270°$

$90° \le x \le 270°$

3 Jia has to solve the equation

$$2 - 2\sin^2\theta = \cos\theta$$

where $-180° \le \theta \le 180°$

Jia's working is as follows:

$$2 - 2(1 - \cos^2\theta) = \cos\theta$$

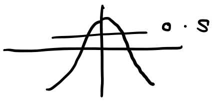
$$2 - 2 + 2\cos^2\theta = \cos\theta$$

$$2\cos^2\theta = \cos\theta$$

$$2\cos\theta = 1$$

$$\cos\theta = 0.5$$

$$\theta = 60°$$



Jia's teacher tells her that her solution is incomplete.

3 (a) Explain the **two** errors that Jia has made.

[2 marks]

1 She divided by $\cos\theta$, missing $\cos\theta = 0$ as a solution

2 Her only solution was $60°$ when she could've also found $-60°$ as a solution.

3 (b) Write down all the values of θ that satisfy the equation

$$2 - 2\sin^2\theta = \cos\theta$$

where $-180° \le \theta \le 180°$

[2 marks]

$2\cos^2\theta - \cos\theta = 0$

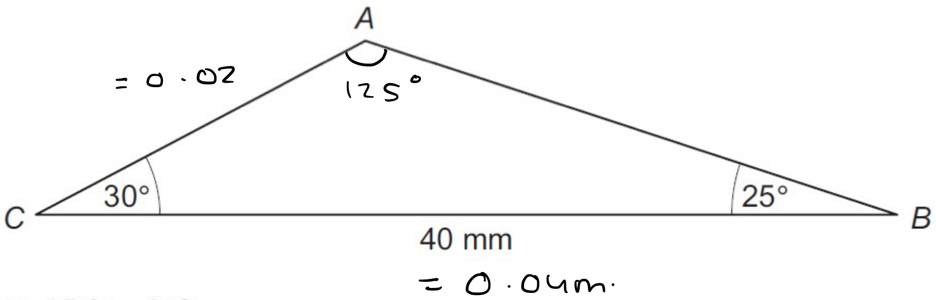
$\cos\theta(2\cos\theta - 1) = 0$

$\cos\theta = 0$ $\quad | \quad \cos\theta = \frac{1}{2}$

$\theta = -90°, 90°$ $\quad | \quad \theta = -60°, 60°$

$\therefore$ all solutions $\theta: -90°, -60°, 60°, 90°$

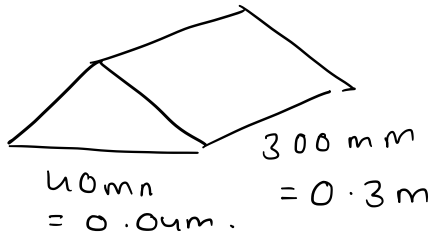
5 A triangular prism has a cross section *ABC* as shown in the diagram below.



Angle $ABC = 25°$

Angle $ACB = 30°$

$BC = 40$ millimetres.



The length of the prism is 300 millimetres.

Calculate the volume of the prism, giving your answer to three significant figures.

[4 marks]

ANGLE CAB = 180 − 30 − 25 = 125

$\frac{0.04}{\sin 125} \times \sin 25 = b = 0.0206368653 9$

Volume $= \frac{1}{2} ab \sin C \times L$

$= \frac{1}{2}(0.04)(0.02\ldots)(\sin 30)\ 0.3$

$\approx 6.19 \times 10^{-5}\ m^3$

4 Find all the solutions of

$$9\sin^2 x - 6\sin x + \cos^2 x = 0$$

where $0° \le x \le 180°$

Give your solutions to the nearest degree.

Fully justify your answer.

[4 marks]

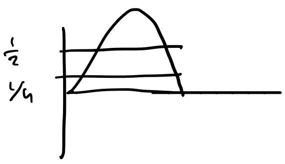
$9\sin^2 x - 6\sin x + (1 - \sin^2 x) = 0$

$8\sin^2 x - 6\sin x + 1 = 0$

let $\sin x = y$

$8y^2 - 6y + 1 = 0$

$(2y - 1)(4y - 1) = 0$



$2y - 1 = 0$ $\quad$ $4y - 1 = 0$

$y = \frac{1}{2}$ $\quad$ $y = \frac{1}{4}$

$\therefore \sin x = \frac{1}{2}$ or $\sin x = \frac{1}{4}$

$x = 30, 150$ $\quad$ $x = 14.5, 165.5$
(1dp) (1dp)

$\therefore x : 14.5°, 30°, 150°, 165.5°$

$x : 15°, 30°, 150°, 166°$

4 Solve the equation $\tan^2 2\theta - 3 = 0$ giving all the solutions for $0° \leq \theta \leq 360°$

[4 marks]

$$\frac{\sin^2 2\theta}{\cos^2 2\theta} - 3 = 0$$

$$\sin^2 2\theta - 3\cos^2 2\theta = 0$$

$$1 - \cos^2 2\theta - 3\cos^2 2\theta = 0$$

$$1 = 4\cos^2 2\theta$$

$$\pm\sqrt{\frac{1}{4}} = \cos 2\theta$$

$\cos 2\theta = +\sqrt{\frac{1}{4}}$ OR $\cos 2\theta = -\sqrt{\frac{1}{4}}$

$2\theta = 60$

$\theta = 30$

$2\theta = 300$

$\theta = 150$

$2\theta = 120$

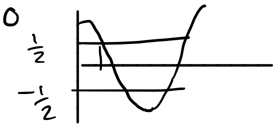
$\theta = 60$

$2\theta = 240$

$\theta = 120$

$\therefore \theta: 30°, 120°, 150°, 240°$

COS GRAPH



10 On 18 March 2019 there were 12 hours of daylight in Inverness.

On 16 June 2019, 90 days later, there will be 18 hours of daylight in Inverness.

Jude decides to model the number of hours of daylight in Inverness, N, by the formula

$$N = A + B\sin t°$$

where t is the number of days after 18 March 2019.

10 (a) (i) State the value that Jude should use for A.

[1 mark]

12

$12 = A + B\sin 0$

$12 = A$

10 (a) (ii) State the value that Jude should use for B.

[1 mark]

6

$18 = 12 + B\sin 90$

$18 = 12 + B$

$6 = B$

10 (a) (iii) Using Jude's model, calculate the number of hours of daylight in Inverness on 15 May 2019, 58 days after 18 March 2019.

[1 mark]

$N = 12 + 6\sin 58 = 17.088... = 17$ hours

10 (a) (iv) Using Jude's model, find how many days during 2019 will have at least 17.4 hours of daylight in Inverness.

[4 marks]

$N \geq 17.4$

$12 + 6\sin t \geq 17.4$

$6\sin t - 5.4 \geq 0$

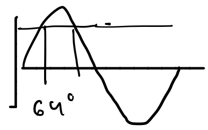
$6\sin t \geq 5.4$

$\sin t \geq 0.9$

$t \geq 64.158$

can't have 0.156

$\therefore 65 \leq t$



$t \leq 180 - 64.15\ldots$

$t \leq 115.84 \leq 116$

$116 - 65 = 51$

51 days will have more than 17.4 hours of daylight.

10 (a) (v) Explain why Jude's model will become inaccurate for 2020 and future years.

[1 mark]

Because the climate is changing due to global warming

10 (b) Anisa decides to model the number of hours of daylight in Inverness with the formula

$$N = A + B\sin\left(\frac{360}{365}t\right)^{\circ}$$

Explain why Anisa's model is better than Jude's model.

[1 mark]

as she uses the actual number of days in a year combined with the degrees to get a more accurate result.

6 (a) (i) Show that $\cos\theta = \frac{1}{2}$ is one solution of the equation

$$6\sin^2\theta + 5\cos\theta = 7$$

[2 marks]

$6(1 - \cos^2\theta) + 5\cos\theta = 7$

$6 - 6\cos^2\theta + 5\cos\theta = 7$

$0 = 6\cos^2\theta - 5\cos\theta + 1$

let $x = \cos\theta$

$0 = 6x^2 - 5x + 1$

$\therefore (2x - 1)(3x - 1) = 0$

$(2\cos\theta - 1)(3\cos\theta - 1) = 0$

$\downarrow$

$2\cos\theta = 1$

$\therefore \cos\theta = \frac{1}{2}$ as required

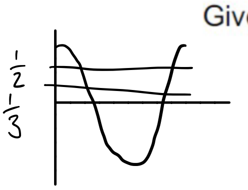
6 (a) (ii) Find all the values of θ that solve the equation

$$6\sin^2\theta + 5\cos\theta = 7$$

for $0° \leq \theta \leq 360°$

Give your answers to the nearest degree.

[2 marks]



$\cos\theta = \frac{1}{2}$ or $\cos\theta = \frac{1}{3}$

$\theta = 60$ $\quad$ $\theta = 70.5$ (1dp)

$\theta = 300$ $\quad$ $\theta = 289.5$ (1dp)

6 (b) Hence, find all the solutions of the equation

$$6\sin^2 2\theta + 5\cos 2\theta = 7$$

for $0° \leq \theta \leq 360°$

Give your answers to the nearest degree.

[2 marks]

2θ : $60°, 71°, 290°, 300°, 420, 431, 650, 660$

$\theta = 30, 36, 145, 150, 210, 216, 325, 330$ $\quad 0.066987$

9 It is given that $\cos 15° = \frac{1}{2}\sqrt{2+\sqrt{3}}$ and $\sin 15° = \frac{1}{2}\sqrt{2-\sqrt{3}}$

Show that $\tan^2 15°$ can be written in the form $a + b\sqrt{3}$, where a and b are integers.

Fully justify your answer.

[3 marks]

$$\tan^2 15 = \frac{\sin^2 15}{\cos^2 15} = \frac{\left(\frac{1}{2}\sqrt{2-\sqrt{3}}\right)^2}{\left(\frac{1}{2}\sqrt{2+\sqrt{3}}\right)^2} = \frac{\frac{2-\sqrt{3}}{4}}{\frac{2+\sqrt{3}}{4}} = \frac{2-\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{(2-\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{4 - 4\sqrt{3} + 3}{4 - 3}$$

$$= 7 - 4\sqrt{3}$$

$a = 7,\ b = 4 \leftarrow$ $= 7 - 4\sqrt{3}$